Photon-Photon Interaction in a Photon Gas

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Abstract

Using the effective Lagrangian for the low energy photon-photon interaction the lowest order photon self energy at finite temperature and in non-equilibrium is calculated within the real time formalism. The Debye mass, the dispersion relation, the dielectric tensor, and the velocity of light following from the photon self energy are discussed. As an application we consider the interaction of photons with the cosmic microwave background radiation.

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Photon-photon scattering has been considered already a long time ago [1]. To lowest order QED perturbation theory it is caused by the so-called box diagram, which contains an electron loop. For low energy photons, i.e. for center of mass energies below the threshold for e^+-e^- pair creation, an effective Lagrangian for the photon-photon interaction has been derived by integrating out the electrons [2],

$$\mathcal{L}_I = a\,(F_{\mu\nu}F^{\mu\nu})^2 + b\,F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu}, \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor and

$$a = -\frac{5\alpha^2}{180 m_e^4}, \qquad\qquad b = \frac{7\alpha^2}{90 m_e^4} \tag{2}$$

with the fine structure constant $\alpha \simeq 1/137$ and the electron mass m_e. This effective Lagrangian containing an effective 4-photon interaction describes the deviation from the classical Maxwell theory by quantum effects. The 4-photon vertex in momentum space following from this Lagrangian has been derived only recently [3]. The coupling constant corresponding to this vertex is of the order α^2/m_e^4.

The lowest order photon self energy $\Pi_{\mu\nu}$, which is given by the tadpole diagram of Fig.1, vanishes at zero temperature after dimensional regularization [3]. However, at finite temperature tadpole diagrams lead to a finite result [4]. The in-medium photon self energy determines the Debye screening, the photon dispersion relations, and the dielectric functions of the system. Some of the results presented here are already discussed in the literature [5,6]. Here we want to treat the photon self energy and its consequences in a systematic and comprehensive way starting from the real time formalism, which also allows an extension to non-equilibrium situations. As an application we consider the influence of the cosmic microwave background radiation on low energy photons. Although medium effects of the photon gas are expected to be very small due to the extremely weak photon-photon coupling at low energies, these effects might be interesting after all since the properties of the photons are experimentally very well known. For example there are very restrictive upper limits for the photon mass [7], namely $m_\gamma < 2 \times 10^{-16}$ eV measured in the laboratory and $m_\gamma < 10^{-27}$ eV using arguments about the galactic magnetic field. These upper limits follow from searching for violations of the Maxwell theory, in particular of the Coulomb law [8]. Hence these limits should be compared to the Debye mass caused by the background of thermal particles. It should be noted that the Debye mass, which follows from the photon self energy, does not violate gauge symmetry of QED.

Mass effects due to charged particles of the thermal background, of which the electron is the lightest, are suppressed exponentially by a factor $\exp(-m_e/T)$, where $T = 2.7$ K is the temperature of the background [9]. This argument, however, does not hold in the case of the photon-photon interaction according to (1), since the electrons, which have been integrated out, come from vacuum polarization. Therefore it appears to be worthwhile to reconsider the effective photon mass due to the cosmic background radiation. Further interesting quantities following from the photon self energy are the dispersion relations of photons in a thermal photon gas and its dielectric function, related to the index of refraction and the velocity of light.

Using the notation $P = (p_0, \mathbf{p})$ and $p = |\mathbf{p}|$ the retarded photon self energy according to Fig.1 reads

$$\Pi_{\mu\nu}(P) = -\frac{1}{2} \int \frac{d^4Q}{(2\pi)^4}\, D^{\rho\sigma}(Q)\, \Gamma_{\rho\mu\sigma\nu}(Q,P,Q,P). \tag{3}$$

where $\Gamma_{\rho\mu\sigma\nu}$ is the effective 4-photon vertex and the factor 1/2 a symmetry factor associated with the tadpole diagram. Adopting the real time formalism in the Keldysh representation [10] the photon propagator in a general covariant gauge with gauge parameter ξ reads

$$D^{\rho\sigma}(Q) = -\left(g^{\rho\sigma} - \xi \frac{Q^\rho Q^\sigma}{Q^2}\right) \frac{1}{2} \left[D_R(Q) + D_A(Q) + D_F(Q)\right]. \tag{4}$$

The retarded (R), advanced (A), and symmetric (F) propagators are given by

$$\begin{aligned} D_{R,A}(Q) &= \frac{1}{Q^2 \pm i\,\mathrm{sgn}(q_0)\varepsilon}, \\ D_F(Q) &= -2\pi i\,[1 + 2n_B(Q,x)]\,\delta(Q^2) \end{aligned} \tag{5}$$

whith the non-equilibrium photon distribution n_B depending on the momentum and the space-time coordinate. In equilibrium it reads $n_B^{\mathrm{eq}} = 1/[\exp(|q_0|/T) - 1]$. In the following we restrict ourselves to isotropic momentum distributions[1], i.e. $n_B = n_B(q_0, q, x)$. Then there are only two independent components of $\Pi_{\mu\nu}$, which depend on p_0 and p [4]. For these components we choose

$$\begin{aligned} \Pi_L(p_0,p) &= \Pi_{00}(P), \\ \Pi_T(p_0,p) &= \frac{1}{2}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) \Pi_{ij}(P). \end{aligned} \tag{6}$$

It should be noted that the longitudinal component is sometimes defined differently [4].

Since the zero temperature contributions vanish we find

$$\begin{aligned} \Pi_L(p_0,p) &= -\frac{i}{2} \int \frac{d^4Q}{(2\pi)^3}\, n_B(q_0,q,x)\, \delta(Q^2)\, \Gamma^\rho{}_{0\rho 0}(Q,P,Q,P), \\ \Pi_T(p_0,p) &= -\frac{i}{4}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) \int \frac{d^4Q}{(2\pi)^3}\, n_B(q_0,q,x)\, \delta(Q^2)\, \Gamma^\rho{}_{i\rho j}(Q,P,Q,P). \end{aligned} \tag{7}$$

Adopting the expression for the 4-photon vertex given in Ref. [3], where some factors of 2 [12] and the sign are corrected, we obtain

$$\begin{aligned} \Gamma^\rho{}_{0\rho 0}(Q,P,Q,P)\Big|_{Q^2=0} &= 16i\,(4a+3b)\,[(\mathbf{p}\cdot\mathbf{q})^2 - p^2 q^2], \\ \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) \Gamma^\rho{}_{i\rho j}(Q,P,Q,P)\Big|_{Q^2=0} &= -16i\,(4a+3b)\,(p_0^2 + p^2)\,q^2 \left[1 + \frac{(\mathbf{p}\cdot\mathbf{q})^2}{p^2 q^2}\right]. \end{aligned} \tag{8}$$

In (7) the gauge fixing parameter ξ does not appear since $Q^\rho Q^\sigma \Gamma_{\rho\mu\sigma\nu}(Q,P,Q,P) = 0$ as can be shown explicitly. Hence the lowest order photon self energy is gauge invariant.

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[1] The anisotropic case has been discussed in Ref. [11] for QED and QCD.

Inserting (8) into (7) we end up with the final result

$$\begin{aligned}\Pi_L(p_0,p) &= -\gamma\, p^2,\\ \Pi_T(p_0,p) &= -\gamma\,(p_0^2+p^2),\end{aligned} \tag{9}$$

where

$$\gamma = \frac{44}{135\pi^2}\,\frac{\alpha^2}{m_e^4}\int_0^\infty dq\, q^3\, n_B(q,x). \tag{10}$$

In equilibrium (10) reduces to $\gamma = (44\pi^2/2025)\,\alpha^2\,(T/m_e)^4$. Then (9) agrees apart from the sign for Π_T with Ref. [6].

Now we want to discuss the physical consequences of our result. First we consider Debye screening in the photon gas. It has been argued that there is no Debye mass due to the photon-photon interaction since the corresponding vertex vanishes if one of the external legs has zero momentum [5]. However, this argument is based on an inconsistent definition for the Debye mass

$$m_D^2 = \Pi_L(p_0=0, p\to 0), \tag{11}$$

where $p = |\mathbf{p}|$. This definition can lead to gauge dependent results and is not renormalization-group invariant [13]. Instead of (11) the Debye mass should be determined self consistently from the pole of the longitudinal photon propagator, i.e. from [13]

$$m_D^2 - \Pi_L(p_0=0, p^2=-m_D^2) = 0. \tag{12}$$

Only if $\Pi_L(p_0=0,p)$ does not depend on p, the definition (11) agrees with (12). Although this is not the case here, the Debye mass following from (9) and (12) vanishes also. This means that the photon-photon interaction in a photon gas does not lead to a screening of the Coulomb potential. Of course, there is also no static magnetic screening.

As another point we mention that the tadpole self energy of Fig.1 has no imaginary part, i.e. neither real nor virtual photons are damped to this order. In QED damping arises from the box diagram only above the threshold for electron-positron pair production. This effect, however, is not included in the effective theory (1) for low energy photons. Damping will be present in the effective theory at the two-loop level (sunset diagram) corresponding to photon-photon scattering.

Next we discuss the photon dispersion relations in the photon gas, which follow from the pole of the resummed photon propagator. In Coulomb gauge the resummed propagator is given by

$$\begin{aligned}D_L^{-1}(p_0,p) &= p^2 - \Pi_L(p_0,p) = (1+\gamma)\,p^2,\\ D_T^{-1}(p_0,p) &= p_0^2 - p^2 - \Pi_T(p_0,p) = (1+\gamma)\,p_0^2 - (1-\gamma)\,p^2.\end{aligned} \tag{13}$$

Whereas there is no dispersion relation for longitudinal photons, i.e., there are no plasmons in the photon gas, the dispersion relation of the transverse (physical) photons is modified compared to the vacuum. It is given by

$$\omega(p) = \sqrt{\frac{1-\gamma}{1+\gamma}}\,p \simeq (1-\gamma)\,p, \tag{14}$$

i.e., the dispersion is located below the light cone $\omega < p$ and the plasma frequency $\omega_{pl} = \omega(p = 0)$ vanishes. The phase velocity $v_p = \omega/p \simeq 1 - \gamma$ is identical to the group velocity $v_g = \partial\omega/\partial p$ and smaller than the speed of light in the vacuum. The result agrees with [14], where it has been derived in QED. Note also that the phase as well as the group velocity are independent of the momentum, i.e., there is no dispersion.

Finally we turn to the dielectric tensor. In an isotropic medium there are only two independent components of the dielectric tensor, for which we choose the longitudinal and the transverse dielectric functions, related to the photon self energy via [15]

$$\begin{aligned} \epsilon_L(p_0, p) &= 1 - \frac{\Pi_L(p_0, p)}{p^2} = 1 + \gamma, \\ \epsilon_T(p_0, p) &= 1 - \frac{\Pi_T(p_0, p)}{p_0^2} = 1 + \gamma\, \frac{p_0^2 + p^2}{p_0^2}. \end{aligned} \tag{15}$$

Since there is no direction preferred for $\mathbf{p} = 0$ [16], the longitudinal and transverse dielectric functions coincide in this limit, $\epsilon_L(p_0, p = 0) = \epsilon_T(p_0, p = 0) = 1 + \gamma$.

The electric permittivity and the magnetic permeability given by [17]

$$\begin{aligned} &\epsilon = \epsilon_L = 1 + \gamma, \\ &\frac{1}{\mu} = 1 + \frac{\Pi_T - p_0^2 \Pi_L / p^2}{p^2} = 1 - \gamma \end{aligned} \tag{16}$$

are independent of p_0 and p. The phase velocity following from [17], related to the index of refraction n,

$$v_p = \frac{1}{n} = \frac{1}{\sqrt{\mu\epsilon}} \simeq 1 - \gamma \tag{17}$$

agrees with the result found from the dispersion relation (14).

Owing to the cosmic microwave background the velocity of light in the Universe is reduced compared to the vacuum. Actually it increases continuously with time as the temperature drops. Today at a temperature of 2.7 K it is given by (17) with $\gamma = 4.7 \times 10^{-43}$. In the early Universe, when the radiation decoupled from matter at a temperature of about 3000 K we had $\gamma = 6.5 \times 10^{-31}$. Although this is probably not a measurable effect, the speed of light is not a constant in our Universe.

Summarizing, we have calculated the photon self energy in an isotropic, non-equilibrium photon gas using the real time formalism. For this purpose we considered the effective Lagrangian for photon-photon interaction and calculated the photon self energy to lowest order perturbation theory using an effective 4-photon vertex in momentum space. As physical consequences of this self energy we showed the absence of Debye and static magnetic screening in the photon gas. Also there are no longitudinal collective modes (plasmons). However, the transverse collective modes exhibit a modified dispersion with a vanishing plasma frequency and a smaller slope compared to the vacuum modes. This results in a reduced, dispersion free velocity of light, which increases during the evolution of the Universe as the temperature of the cosmic microwave background drops. We also determined the dielectric tensor, the

electric permittivity and the magnetic permeability of the photon gas.

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FIGURES

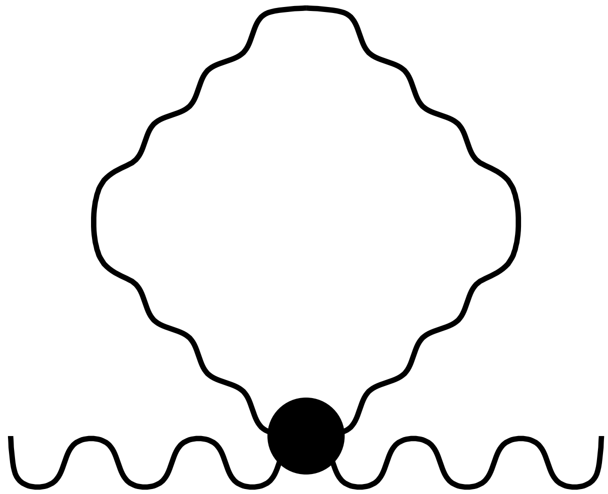

FIG. 1. Lowest order photon self energy in the effective theory for photon-photon interaction. The blob denotes the effective 4-photon vertex given in Ref.[3].